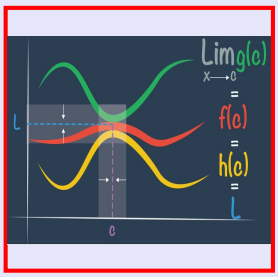
Math 261

Spring 2023

Lecture 42



Feb 19-8:47 AM

class QZ 12.

$f'(x) = 5x\sqrt{x}$ $\quad f(4) = 0$, find $f(x)$.

$f'(x) = 5x^{1} \cdot x^{1/2} = 5x^{\boxed{\frac{3}{2}}}$

$f(x) = 5 \cdot \dfrac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C = \dfrac{5x^{\frac{5}{2}}}{\frac{5}{2}} + C = \boxed{2x^{5/2} + C}$

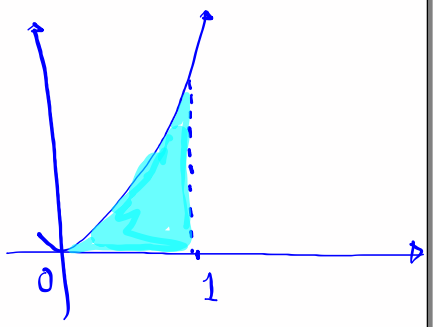
$f(4) = 0 \qquad 2(4)^{5/2} + C = 0 \qquad 2(2)^5 + C = 0$

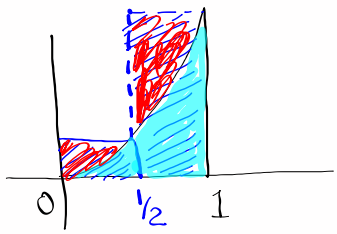
$\boxed{f(x) = 2x^{5/2} - 64} \quad \boxed{f(x) = 2x^2\sqrt{x} - 64} \quad \boxed{C = -64}$ ✓

Apr 27-9:36 AM

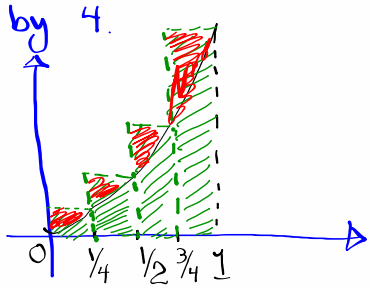
Find the area below $f(x) = x^2$, above x-axis from $x=0$ to $x=1$.

Divide interval $[0,1]$ by 2



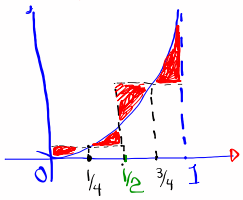


Divide interval $[0,1]$ by 4.

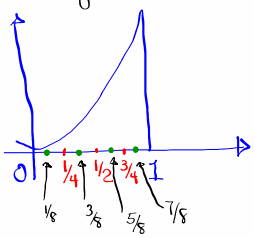


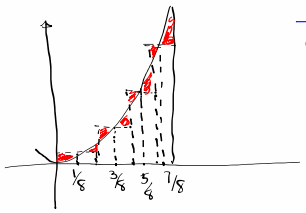
May 1-8:50 AM

Divide interval $[0,1]$ by 2, then take the middle point to get areas.



Do Similar with dividing by 4.





Each area belongs to a rectangle.

$A = LW$

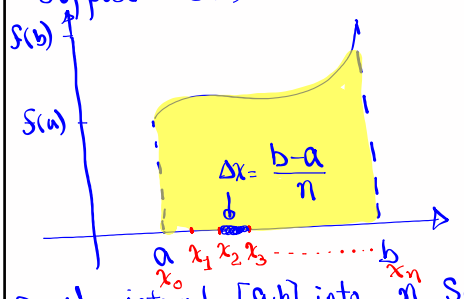
As number of intervals increases, we have more rectangles.

Once we find areas of each rectangle, then add all those areas together, we will find Total area.

May 1-8:58 AM

Find the area below $f(x)$, above x-axis from $x=a$ to $x=b$.

Suppose $f(x) \geq 0$, $f(x)$ is cont. on $[a,b]$



Δx ← Delta

Divide interval $[a,b]$ into n subintervals.

$x_1 = a + 1\Delta x$

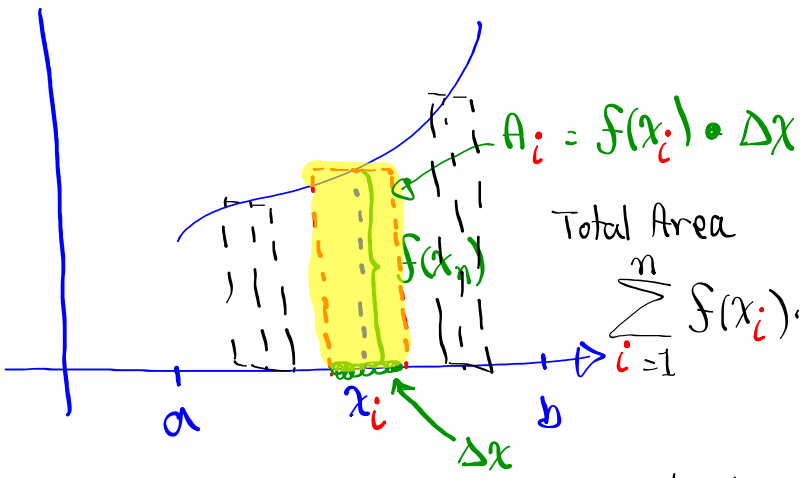
$x_2 = x_1 + 1\Delta x = a + 2\Delta x$

$x_3 = x_2 + 1\Delta x = a + 3\Delta x$

$\vdots$

$x_i = x_{i-1} + 1\Delta x = a + i\,\Delta x$

May 1-9:08 AM



Total Area

$$\sum_{i=1}^{n} f(x_i)\cdot\Delta x$$

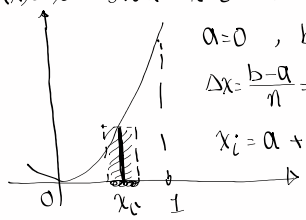
As n gets bigger, we get better result.

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x$$

if this limit exist, we have the total area

May 1-9:15 AM

Find the area above x-axis, below $f(x) = x^2$ from $x=0$ to $x=1$.



$a=0$, $b=1$

$\Delta x = \frac{b-a}{n} = \frac{1-0}{n} = \frac{1}{n}$

$x_i = a + i\Delta x = 0 + i \cdot \frac{1}{n}$

$x_i = \frac{i}{n}$

$A_i = f(x_i)\cdot\Delta x = f\left(\frac{i}{n}\right)\cdot\frac{1}{n} = \left(\frac{i}{n}\right)^2\cdot\frac{1}{n} = \frac{i^2}{n^3}$

Total Area $\sum_{i=1}^{n} f(x_i)\cdot\Delta x = \sum_{i=1}^{n} A_i = \sum_{i=1}^{n}\frac{i^2}{n^3}$

$A = \lim_{n\to\infty}\sum_{i=1}^{n} A_i = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^2}{n^3}$

$= \lim_{n\to\infty}\frac{1}{n^3}\sum_{i=1}^{n} i^2 = \lim_{n\to\infty}\frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$

from precalculus,

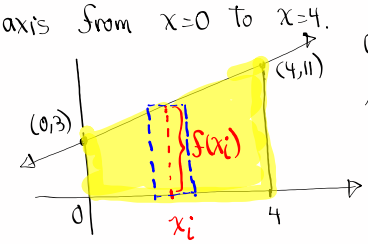
$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)\cdot(2n+1)}{6}$

$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

$= \lim_{n\to\infty}\frac{2n^3 + \text{Junk}}{6n^3} = \boxed{\frac{1}{3}}$

May 1-9:21 AM

Find the area below $f(x) = 2x+3$, above x-axis from $x=0$ to $x=4$.



$a=0$, $b=4$

$\Delta x = \frac{b-a}{n} = \frac{4}{n}$

$x_i = a + i\Delta x$

$= 0 + i\cdot\frac{4}{n}$

$= \frac{4i}{n}$

$A_i = f(x_i)\cdot\Delta x = [2x_i + 3]\cdot\frac{4}{n}$

$= \left[2\cdot\frac{4i}{n} + 3\right]\cdot\frac{4}{n} = \left(\frac{8i}{n} + 3\right)\cdot\frac{4}{n}$

$\sum_{i=1}^{n} A_i = \sum_{i=1}^{n}\left(\frac{8i}{n}+3\right)\cdot\frac{4}{n}$

$A = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{8i}{n}+3\right)\cdot\frac{4}{n} = \lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n}\left(\frac{8i}{n}+3\right)$

$= \lim_{n\to\infty}\frac{4}{n}\cdot\left[\sum_{i=1}^{n}\frac{8i}{n} + \sum_{i=1}^{n} 3\right]$

$= \lim_{n\to\infty}\frac{4}{n}\cdot\left[\frac{8}{n}\sum_{i=1}^{n} i + 3\sum_{i=1}^{n} 1\right]$

May 1-9:36 AM

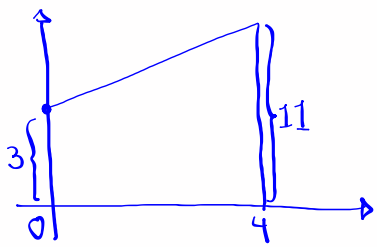
$$= \lim_{n\to\infty} \frac{4}{n} \cdot \left[\frac{8}{n} \sum_{i=1}^{n} i + 3 \sum_{i=1}^{n} 1 \right]$$

$$= \lim_{n\to\infty} \frac{4}{n} \left[\frac{8}{n} \cdot \frac{n(n+1)}{2} + 3 \cdot n \right]$$

$$= \lim_{n\to\infty} \left(\frac{32n^2 + \text{junk}}{2n^2} + 12 \right) = \frac{32}{2} + 12$$

$\frac{32}{2}$

$$= \boxed{28}$$



Trapezoid

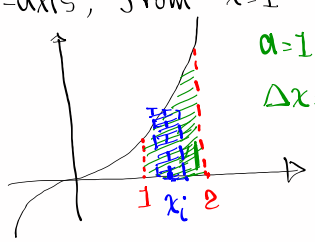
$h = 4$, $b = 3$, $B = 11$

$$A = \frac{(b+B)\cdot h}{2} = \frac{(3+11)\cdot 4}{2}$$

$$= \boxed{28}$$

May 1-9:44 AM

Find the area below $f(x) = x^3$, above x-axis, from $x=1$ to $x=2$.



$a = 1$, $b = 2$

$$\Delta x = \frac{b-a}{n} = \frac{1}{n}$$

$$x_i = a + i\Delta x$$

$$= 1 + i \cdot \frac{1}{n} = 1 + \frac{i}{n}$$

$$A_i = f(x_i) \cdot \Delta x = \left(1 + \frac{i}{n}\right)^3 \cdot \frac{1}{n}$$

Recall

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$\left(1 + \frac{i}{n}\right)^3 =$$

$$A_i = \left(1 + \frac{i}{n}\right)^3 \cdot \frac{1}{n}$$

May 1-9:50 AM